Comparison of Eleven Mathematical Models for describing the first Lactation Curve of Holstein Cattle in Turkey

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Abstract: In this study, eleven standard lactation curve models (Incomplete Gamma (WD), Quadratic (Q), Cubic (C), Linear Hyperbolic Function (LH), Inverse Polynomial Function (IP), Mixed Log (MIL), Exponential (WIL), Dhanoa (DH), Cobby and Le Du (CD), Polynomial Regression (AS) and New Model (NM)) were used to predict a typical dairy cow lactation derived as the average daily milk yield of 105 complete first lactations of Holstein-Friesian cows in one herd. Milk yield controls were made daily in this farm. Total milk yield (TMY) was calculated from observation of daily milk yield. TMY was also predicted by using 11 different models. The total milk yields predicted by the models were very close to each other and the differences between observation of TMY were not found statistically significant (P>0.05). The models were found to be adequate for estimation of milk yield.

Determination coefficients (R^2) of the models ranged from 67.15 % to 86.68 %. In comparison of the models, the TMY, peak yield (PY), peak time (PT), persistency (P), mean square prediction error (MSPE), approximation error (ε), reliability criterion for estimating trustworthiness of the determination coefficient (σ), standard error (θ) and Durbin-Watson (DW) values were evaluated together.

Using the AS, WD and new developed NM models accurately predicted the milk yield of Holstein cows.

Key Words: Holstein, Cows, Lactation Curve, Milk Yield, Mathematical Model

Introduction

Turkey has 11.3 million head of cattle and 70 % of them are improved cattle and their crossbreeds. The number of milked animals is 4.2 million and approximately 3 million of them are improved cattle and their crossbreeds. The milk obtained from cows is 11.3 million tons and nearly 86 % of the milk production is produced by improved cattle and their crossbreds. But the lactation milk yields is very low (i.e. for native breeds 1.3, for crossbreeds 2.7 and for improved cattle 3.9 tons) (TurkStat, 2007).

Producers aim to increase milk yield and decrease cost for a profitable dairy cattle production. Persistency is one of the most important factors which determine milk production cost along the lactation. Milk yield begins with calving and reach to highest level between 40 to 70 days and then continues to decrease along the lactation. With decreasing of daily milk yield, the production cost begins to increase from day to day (Gengler, 1996; Koçak and Ekiz, 2006). A mathematical model of the lactation curve provides summary information about dairy cattle production, which is useful in making management and breeding decisions and in simulating a dairy enterprise (Olori et al., 1999). In order to asses plausible forms of lactation curves, milk yield records collected throughout the whole lactation are required. But most of the small and medium sized dairy farms in Turkey still use classical milking systems. Milk yield is generally recorded monthly in these farms. The lactation curve models enable them to evaluate lactation as a whole. So that the lactation curve shape is determined and unbiased comparison methods among animals with incomplete lactation records for genetic evaluation purposes can be practiced (Keown and van Vleck, 1973). Knowledge of the lactation curve allows prediction of total milk production from partial production measured at several test days early in lactation (Goodal and Sprevak, 1985). Animals with a high milk yield potential can be identified by using this information before the whole lactation is completed. Also, lactation curves can be used for prediction of lifetime milk production from early lactation traits (Dalal et al., 2004), culling, assessing nutritional and health status of animals (Duoduet, 1982; Souvant and Fehr, 1975) and evaluating a suitable time to end milking (Chang et al., 2001).

The first mathematical model aimed to describe lactation curves was developed by Brody et al., (1923). It was notified that this model was followed by the models reported out by Sika (1950), Nelder, (1966), Wood (1967), Dave (1971) and Jenkins and Ferrel (1984) (Landete-Castillejos and Gallego, 2000). The Wood model has been used in most lactation curve model studies, because it includes the basic features of lactation curves with only three parameters a, b and c which allow the calculation of average yield, peak yield and peak time, respectively. This has made the Wood model the most widely used function for the description of lactation curves. Most of the alternative models are also based on the Wood model (Cobby and Le Du, 1978; Wilmink, 1987; Papajcsik and Bodero, 1988). However, some mathematical models have been proposed to describe the regular shape of the lactation curve in dairy cows from partial or incomplete data (Neal and Thornley, 1983; Goodal and Sprevak, 1984; Batra, 1986; Morant and Gnanasakthy, 1989; Dijkstra et al., 1997; Olori et al., 1999; Vargas et al., 2000). Also these models provide analysing systemic changes in milk yield caused by environmental factors (Goodall and Sprevak, 1985; Morant and Gnanasakthy, 1989) and determining the milk production characteristics such as persistency (Gengler, 1996), peak yield and time to peak yield (Masselin et al., 1987; Gipson and Grossman, 1990).

The objective of this study was to compare the suitability of WD, WIL, MIL, C, Q, DH, IP, CD, LH, AS and NM models to the first lactation data of Holstein cows.

Materials and Methods

The data of this study was from the first lactation records of 105 Holstein cows raised in a private enterprise in the Karapınar district (37° 42' K, 33° 35' D and 994 m above sea level) of the Konya Province in the Central Anatolia Region of the Turkey. The data were collected from the first lactation records of the cows that gave birth in 2004. They were machine milked twice daily and milking records were started 3th days of lactation. There is a computer-based herd managing program in the enterprise and milk yield controls were made daily. Average lactation length was 312±4.37 days. The experiment was carried out according to guidelines of Selçuk University Faculty of Agriculture located in the Konya Province.

In the study, to explain lactation curves, eleven different empirical mathematical models were used together and compared. These models are as follows:

(1) Incomplete Gamma (WD), (Wood, 1967):

$$Y_{(t)} = at^b e^{-ct}$$

(2) Quadratic (Q), (Dave, 1971):

$$Y_{(t)} = a + bt + ct^2$$

(3) Cubic (C),

$$Y_{(t)} = a + bt + ct^2 + dt^3$$

(4) Exponential (WIL), (Wilmink, 1987):

$$Y_{(t)} = a + be^{-kt} + ct \quad \text{(Which was fitted with the parameter k fixed at 0.61)}$$

(5) Mixed log (MIL), (Guo and Swallve, 1995):

$$Y_{(t)} = a + bt^{1/2} + c \log t$$

(6) Polynomial Regression (AS), (Ali and Schaeffer, 1987):

$$Y_{(t)} = a + bt + ct^2 + d \log t + e \log t^2$$

(7) Cobby and Le Du (CD), (Cobby and Le Du, 1978):

$$Y_t = a - bt - ae^{-ct}$$

(8) Linear Hyperbolic Function (LH), (Bianchini, 1984):

$$Y_{(t)} = a + bt + c(1/t)$$

(9) Inverse Polynomial Function (IP), (Nelder, 1966):

$$Y_{(t)} = t/(a + bt + ct^2)$$

(10) Dhanoa (DH), (Dhanoa, 1981):

$$Y_t = at^{bc}e^{(-ct)}$$

(11) New developed model (NM):

$$Y_{(t)} = at^b e^{-ct - \frac{d}{t}}$$

For all models, Y_t is the observed milk yield at day t,
a: is linked to milk yield at the beginning of lactation,
b: to the ascending phase before peak yield,
c: to the decreasing phase after peak yield,
d: parameters which characterize the shape of the curve
e: is the base of natural logarithm,
which were estimated from a nonlinear regression analysis using the Statistica program. The WIL model has a total of four parameters, with k being exponent; following Wilmink (1987) a fixed value of k was used, which was estimated at 0.61 in a preliminary analysis as the best fitting value for the herd mean data. Subsequently the WIL model was considered as a three parameter curve in analysis of individual animals.

Persistency (P) was calculated as:

$$P (\%) = \frac{\sum_{i=1}^{k}(pi+1)/pi}{k} \times 100$$

Where *pi* is the yield of the record *i* that starts at peak time and *k* is the record number from peak time to the end of lactation (Sturtevant, 1986).

The parameters obtained were used to calculate the predicted yields in the original equations at above. Residuals, defined as the absolute values of the difference between the predicted yield and real data of daily milk yield, were calculated and then the mean square prediction error (MSPE) for each lactation curve fitted was calculated and averaged for each model (Ruiz *et al.*, 2000).

True peak yield (PY) was determined from the test day milk yield means for the 105 cows and true peak time (PT) was determined as the average day on which daily milk yields were at their maximum value. Peak time values of the models were calculated by equalizing the first partial derivations of the functions to zero, and PY values were found for each cow by replacing PT values in the functions.

The Durbin-Watson statistics was used as a measure of first order positive autocorrelation to test whether the residuals were randomly distributed (Grossman and Koops, 1988). DW was calculated for each lactation and models.

Approximation error was calculated as:

$$\varepsilon = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{Y_i - \overline{Y}}{Y_i}\right|$$

Reliability criterion for estimating trustworthiness of the determination coefficient was calculated as:

$$\theta = \frac{R^2}{1-(R^2)^2}\sqrt{n}$$

Standard error was calculated as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \overline{Y})}{n-m}}$$

The models were compared in respect of their MSPE, correlation between yields and residuals (RESC), R^2, TMY, peak yield (PY), peak time (PT), persistency (P), ε, θ and σ.

Results and Discussion

Lactation curve parameters in Holstein cattle were given in Table 1. The parameter a, which expressed the milk yield at beginning was 0.10 in IP model, and ranged from 12.71 to 25.81 among the other models. Estimates of parameter a, were found in this study more higher than estimated for WD, Q, C, LHF and IP models in Brown Swiss cows by Keskin and Tozluca (2004), for Q, C and WIL models in Simmental cows by Çilek and Keskin (2008), for WD, IP and AS models in Holstein-Friesian cows by Olori *et al*., (1999); but less than estimated for WD, MIL and AS models in Simmental cows by Çilek and Keskin (2008), for WD, MIL, WIL and AS models in Brown Swiss cows by Keskin *et al*., (2009), for MIL and WIL models by Olori *et al*., (1999) and very close to value estimated for WIL model in Holstein cows by Dĕdková and Nĕmcová (2003).

Models[*]	Model parameters				
	$a \pm S_{\bar{a}}$	$b \pm S_{\bar{b}}$	$c \pm S_{\bar{c}}$	$d \pm S_{\bar{d}}$	$e \pm S_{\bar{e}}$
WD	15.34±0.428	0.161±0.0081	0.0030±0.00012		
Q	22.04±0.419	0.024±0.0038	-0.0002±0.00001		
C	20.53±0.649	0.062±0.0124	-0.0004±0.00008	0.00000054±0.0000001	
AS	12.71±0.474	-0.640±0.1099	2.1894±0.34652	6.330±0.5275	-1.022±0.1533
WIL	24.96±0.477	-26.064±1.1519	-0.0307±0.00140		
MIL	13.25±0.483	-1.886±0.0831	6.2888±0.25894		
DH	20.38±0.759	-49.468±11.7626	0.0020±0.00017		
LHF	25.81±0.438	-0.034±0.0015	-17.1840±0.74544		
CD	25.44±0.439	0.032±0.0015	18.3727±3.31720		
IP	0.10±0.012	0.034±0.0009	0.0001±0.000005		
NM	17.11±0.704	0.145±0.0118	0.0029±0.00013	0.153±0.0587	

[*]: WD: Incomplete Gamma, Q: Quadratic, C: Cubic, LH: Linear Hyperbolic Function, IP: Inverse Polynomial Function, MIL: Mixed Log, WIL: Exponential, DH: Dhanoa, CD: Cobby and Le Du, AS: Polynomial Regression and NM: New Model

Table 1. Estimates of the model parameters and their standard errors of eleven models

The highest estimate of parameter b was fixed in WD model, but the lowest estimate was fixed in DH model. Estimates of parameter c were ranged from -17.1840 to 18.3727.

Estimated lactation curve parameters for this herd were generally different from the previous studies. It may be due to raising in different environmental conditions or a result of management and administration in different ways in terms of milk production. On the other hand it is well known that the Holstein breed is more productive in temperate climatic zones and its milk production capacity may change by geographical regions.

The parameter a expressing milk yield at the beginning and the b parameter indicating the speed of curve increase must be higher and the c parameter meaning the speed of curve decrease must be lower in order to obtain more milk production.

The lactation curves of Holstein cows are given in Figures 1. As seen in this figure, fit lines of WD, AS and NM models are very close to the observed values. The total milk yield predicted by different models is very close to observed total milk yield, and the differences between them, were not significant (P>0.05).

| Models | TMY (l) | PY (l) | PT (day) | Goodness of fit statistics | | | | | | | |
				P (%)	R^2 (%)	MSPE	RESC	ε	σ	Θ	DW
WD	6407±150 [ns]	24.50±0.368 [bc]	72.84±17.20 [a]	99.7±0.01 [ns]	76.17±0.014	5.95±0.321	-0.13	14.00	2.47	45.52	0.849
Q	6370±148 [ns]	22.04±0.414 [d]	0.00±0.000 [d]	99.5±0.01 [ns]	75.23±0.013	6.11±0.300	0.30	13.71	2.43	39.61	0.776
C	6338±145 [ns]	20.53±0.649 [d]	0.00±0.000 [d]	99.6±0.03 [ns]	77.87±0.011	5.75±0.712	0.31	14.12	2.55	41.70	0.804
AS	6370±148 [ns]	23.96±0.419 [c]	56.14±3.267 [a]	99.8±0.03 [ns]	80.65±0.011	4.90±0.256	0.00	12.74	2.18	54.36	0.949
WIL	6370±148 [ns]	24.90±0.448 [bc]	10.16±0.15 [c]	99.9±0.05 [ns]	86.68±0.171	7.04±0.370	0.17	15.33	2.59	34.74	0.716
MIL	6370±148 [ns]	25.29±1.078 [bc]	62.79±10.82 [a]	99.8±0.04 [ns]	75.25±0.015	5.99±0.329	-0.17	14.29	2.39	44.71	0.833
DH	6372±148 [ns]	24.46±0.358 [bc]	-46.17±11.66 [d]	99.6±0.01 [ns]	63.59±0.025	7.90±0.542	0.61	15.90	2.69	36.90	0.737
LHF	6370±148 [ns]	24.71±0.366 [bc]	23.69±0.771 [c]	99.9±0.38 [ns]	72.22±0.016	6.57±0.356	0.04	14.95	2.51	39.01	0.775
CD	6364±148 [ns]	25.08±0.395 [bc]	13.40±1.288 [c]	99.8±0.03 [ns]	69.93±0.019	6.88±0.370	0.13	15.13	2.57	36.77	0.732
IP	6335±147 [ns]	22.88±0.424 [b]	29.31±1.287 [bc]	99.9±0.02 [ns]	67.01±0.019	7.57±0.418	-0.35	15.78	2.70	31.69	0.673
NM	6309±159 [ns]	24.65±0.366 [bc]	45.67±2.89 [ab]	99.6±0.03 [ns]	76.88±0.014	5.67±0.315	-0.17	13.93	2.33	47.30	0.877
Really	6369±149 [ns]	28.72±0.422 [a]	69.38±4.85 [a]	99.9±0.03 [ns]							

[a, b, c, d]: The means within columns with different superscript are significantly different at P<0.01

[ns]: not significant.

Table 2. Comparison of the models for estimating total milk yield (TMY), peak yield (PY), time to peak yield (PT), Persistency (P), Correlation between yields and residuals (RESC) and goodness-of-fit statistics (R^2 and MSPE values)
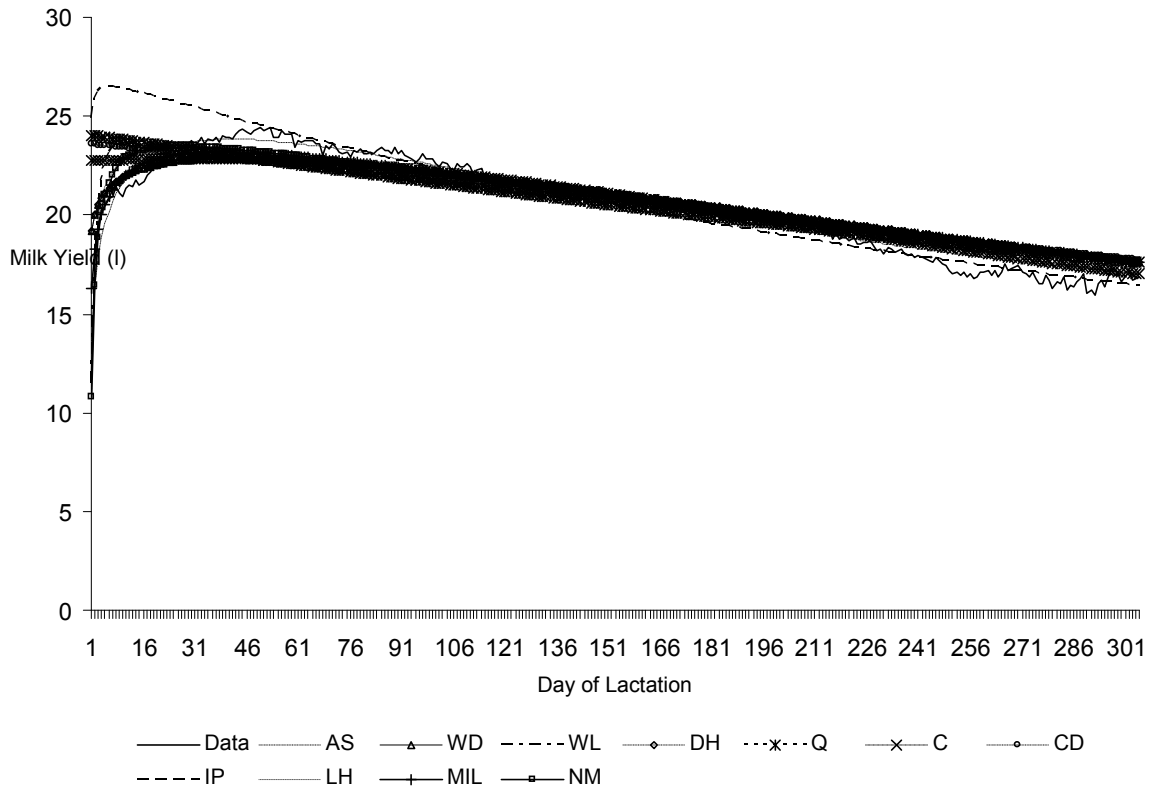
Figure 1. Shape of lactation curve according to the models

Total milk yield (TMY), peak yield (PY), time to peak yield (PT), Persistency (P), Correlation between yields and residuals (RESC) and goodness-of-fit statistics (R^2 and MSPE values) of the models are given in Table 2. In this study, the differences between estimated and observed peak yields were found significant (P<0.01). But the differences between Q and C models' peak yields were not significant and very close predictions obtained with WD, WIL, MIL, DH, LHF, CD, IP and NM models. However the really peak yield was found as 28.72, all models underestimated the peak yield. Generally the curves of real lactation data based on daily milk yields are very fluctuant. But the estimated yields of the models are not fluctuant and also the estimated peak yields do not show sharply increases as seen in real data. On the other hand it is expected that the real lactation data of well managed herds do not fluctuate.

The peak time obtained from actual milk yield and the predicted PT from WD, AS and MIL models were very close and the differences between them, were not significant (P>0.05). But, the peak time of DH model was negative and the peak times of Q and C models were estimated to be only zero. It is likely caused by the decreasing curves from the beginning to the end of lactation for DH, Q and C models estimated by present data as seen in Figure 1.

Persistency (P) values among all models were very close to each other and the differences between them, were not significant (P>0.05).

The P values, were found in this study were higher than values predicted for WD, Q, C, LHF and IP models in Brown Swiss cows by Keskin and Tozluca (2004), for Q, C, WD, MIL, AS and WIL models in Simmental cows by Çilek and Keskin (2008).

Higher determination coefficients for the used models show good fitting level of independent variables for explaining dependent variables. For all models R^2 value were estimated between 63.59 % and 86.68 %, it was obtained the lowest value in DH (63.59) model, the highest in WIL (86.68) model. The best fitness was obtained with WIL, it was followed by AS, but DH fitted worst. The lowest MSPE values were fixed in AS model, then in WD model. The highest MSPE value was fixed in DH model. The R^2 in this study, were lower than R^2 values, were notified for WD, Q, C, LHF and IP models in Brown Swiss cows by Keskin and Tozluca (2004), for Q, C, WD, MIL, AS and WIL models in Simmental cows by Çilek and Keskin (2008), for WD, IP, WIL, MIL and AS models in Holstein-Friesian cows by Olori *et al*., (1999).

MSPE values were higher than values, notified for Q, C, WD, MIL, AS and WIL models in Simmental cows by Çilek and Keskin (2008).

The lowest ε values were realized in AS model and it was followed by Q, NM and WD models. The same condition was found for σ values, too. The highest θ values were estimated in AS model, it was followed by NM and WD models. Autocorrelation values for all models were close to zero, indicating positive autocorrelation which may pose problems with statistical inferences about the models.

Correlations between the residuals and observed milk yield (RESC) ranged between -0.35 (IP) to 0.61 (DH) for all models. Though estimated residuals generally increased with observed yields, there are a little except for C, Q and DH. High daily yields being most difficult to predict while very low yields also caused problems (Olori *et al*., 1999).

Conclusion

The TMY, PY, PT and P values in AS, WD and NM models were found very close to actual values, but MSPE, RESC, ε and σ values were the lowest than actual. The highest R^2 and θ values were found in these models.

As the result of assessing of TMY, PY, PT, P, R^2, MSPE, RESC, ε, σ, θ and DW statistics together, it can be said that the using AS, WD and new developed NM models make possible of predicting milk yields, close to actual values in Holstein cows at first lactation.

Acknowledgments

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