Control of a chaotic finance system with passive control

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Abstract

In this paper, complicated dynamical behavior of a finance system is investigated. The change in behavior of finance system from stable behavior to chaotic behavior is shown with varying some system parameters. In addition, chaotic finance system with passive control is considered and the stability of the controlled system is investigated. In order to control the chaos in finance system, the controller is designed based on passive control technique. Designed controller is applied to the chaotic finance system for stabilization of system. After controller is added to the system, the change in behavior of finance system from chaotic behavior to stable behavior is shown with passive control.

Keywords: Chaotic finance system, chaos control, passive control

1. INTRODUCTION

In 1963, Lorenz found the first chaotic attractor, which is named as Lorenz chaotic system, in a three dimensional autonomous system when he studied atmospheric convection (Lorenz, 1963). After Lorenz, many different chaotic systems are proposed in the past few decades such as Rössler system (OE, 1976), Chen system (G. Chen, 1999), Lü system (C.X. Liu, 2004) and finance chaotic system (Guoliang Cai, 2007). However, when chaotic behavior is sometimes undesirable, the chaotic behavior of system should be controlled. So, many methods and techniques have been developed to control the chaotic systems such as OGY method (E. Ott, 1990), sliding mode control (K. Konishi, 1998), adaptive control (Y.Zeng, 1997), and passive control (Yu, 1999; X. Chen, 2010; S. Emiroğlu, 2010).

In this paper, we study the complicated dynamic behavior and control of chaos in a nonlinear finance chaotic system which was investigated by reference (Guoliang Cai, 2007). The state equations of chaotic finance system are written below Eq 1. (Guoliang Cai, 2007)

$$\begin{aligned} \dot{x} &= z + (y - a)x \\ \dot{y} &= 1 - by - x^2 \\ \dot{z} &= -x - cz \end{aligned} \qquad (1)$$

where variable x represents the interest rate in the model; variable y represents the investment demand and variable z is the price exponent. The parameter a is the saving. b is the per-investment cost. c is the elasticity of demands of commercials. And they are positive constants.

Mathematical model of a finance system is constructed by using Matlab-Simulink program as shown in Figure 1.

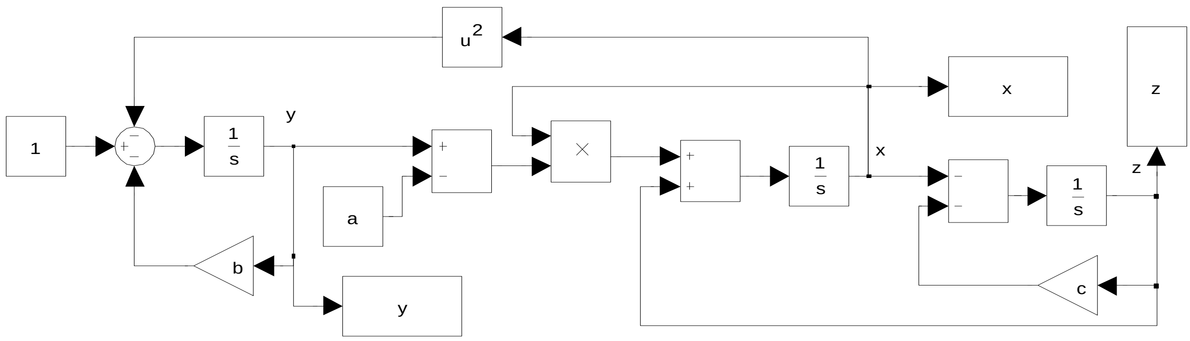

Figure 13 Matlab-Simulink model of finance system

By using Matlab - Simulink model of finance system, chaotic time series and phase portraits of the system is shown in Figure 2.

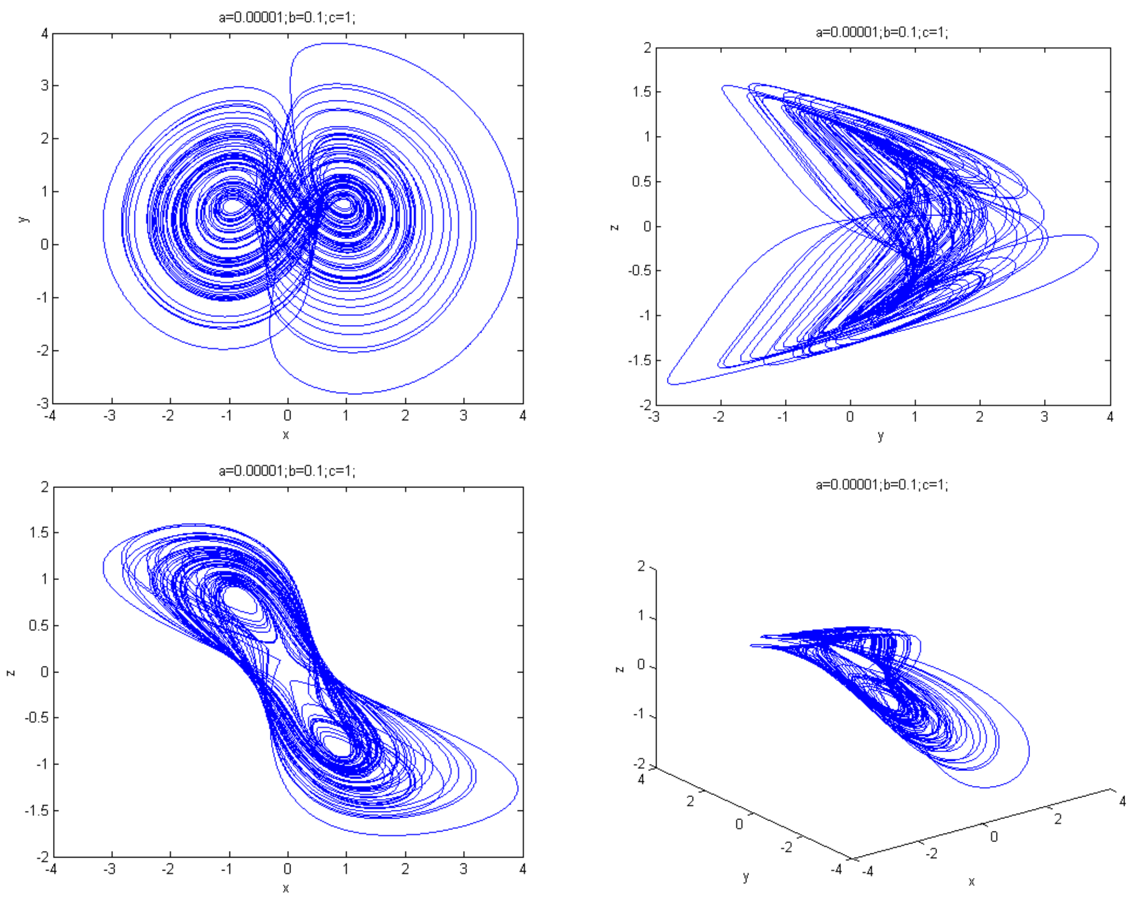

Figure 14 Phase portraits of the system

2. THE THEORY OF PASSIVE CONTROL

Consider a nonlinear system (2) modelled by ordinary differential equation with input vector u(t) and output vector y(t) (Yu, 1999),

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{2}$$

where the state variable $x \in \Re^n$, the input $u \in \Re^m$ and the output $y \in \Re^m$. f(x) and g(x) are smooth vector fields. h(x) is a smooth mapping. We suppose that the vector field f has at least one equilibrium point and without loss of the generality, we assume the equilibrium point x=0.

Definition 1. System (2) is a minimum phase system if Lgh(0) is nonsingular and x=0 is one of the asymptotically stabilized equilibrium points of f(x).

Definition 2. System (2) is passive if the following two conditions are satisfied:

(1) f(x) and g(x) exist and are smooth vector fields, h(x) is also a smooth mapping.

(2) For any $t \geq 0$, there is a real value β that satisfies the inequality

$$\int_0^t u^T(\tau)y(\tau)d\tau \geq \beta, \tag{3}$$

or there are real values β and $\rho \geq 0$ that satisfy the inequality

$$\int_0^t u^T(\tau)y(\tau)d\tau + \beta \geq \int_0^t \rho y^T(\tau)y(\tau)d\tau, \tag{4}$$

When we let $z = \Phi(x)$ system (2) can be changed into the following generalized form

$$\begin{cases} \dot{z} = f_0(z) + p(z,y)y, \\ \dot{y} = b(z,y) + a(z,y)u, \end{cases} \tag{5}$$

where $a(z,y)$ is nonsingular for any (z, y).

If system (2) has relative degree [1,1, ...] at x = 0 and system (1) is a minimum phase system, then system (5) will be equivalent to a passive system and will be asymptotically stable at equilibrium points through the local feedback control as follows:

$$u = a(z,y)^{-1}[-b^T(z,y) - \frac{\partial W(z)}{\partial z} p(z,y) - \alpha y + v] \tag{6}$$

where $W(z)$ is the Lyapunov function of $f_0(z)$, α is a positive real value, and v is an external signal which is connected to the reference input.

3. CHAOS CONTROL OF CHAOTIC FINANCE SYSTEM

In this section, the control of chaotic system (7) is achieved using passive control theory. The controlled model given by

$$\begin{aligned} \dot{x} &= z + (y-a)x \\ \dot{y} &= 1 - by - x^2 + u \\ \dot{z} &= -x - cz \end{aligned} \tag{7}$$

The controller is designed based on passive control theory (Yu, 1999). The controller is shown in Eq. 8 and also controlled system is written in Eq. 7.

$$u = -1 + y(b-\alpha) + v \tag{8}$$

Time series of system and controlled system are shown in Fig. 3. After the controller is activated at t=300s, the system converges to zero equilibrium point as shown in Fig. 3.

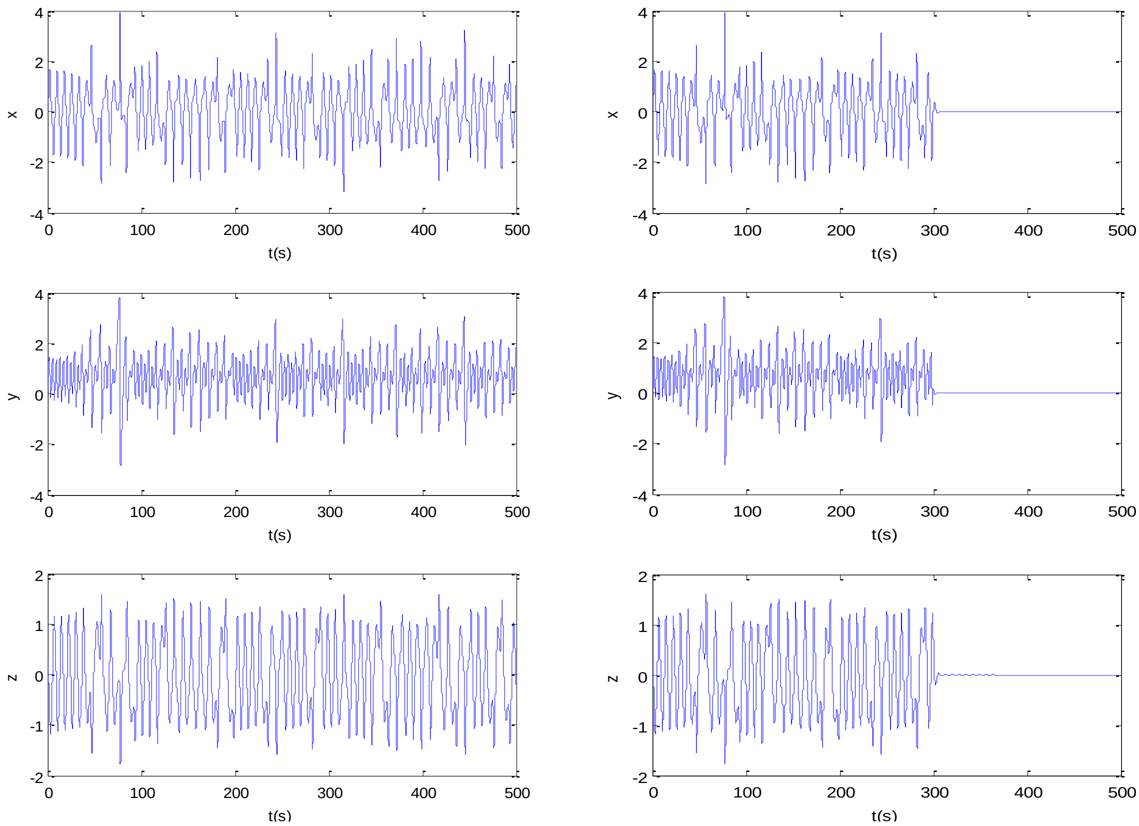

Figure 15 x, y and z time series of (a) system, (b) controlled system

Figure 16 x, y and z time series of (a) system, (b) controlled system

4. CONCLUSION

We investigate chaos control of a 3D chaotic finance system via passive control method in this paper. Based on the passive system theory, passive controller is proposed to realize the global asymptotical stability of the 3D chaotic finance system. Finally, numerical simulations are provided to verify the theoretical analysis and also show that the proposed method works effectively.

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