English for Mathematicians: from Language to Tasks

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Abstract: The author describes the language education system implemented at a faculty preparing professional mathematicians, physicists and IT specialists, paying special attention to the course for mathematicians. The article also presents a profile of a typical attendant of the course based on the theory of learning styles, which together with the stylistic features of the language of mathematics, predetermines the syllabus and appropriate teaching methodology. The author proposes some essential principles on the background of the theory of ESP, EAP and task-based teaching and learning, giving a number of examples from his teaching practice.

Key words: artificiality, curriculum, EAP, ESP, language of mathematics, language of science, linguistic competence, mathematical text, naturalness, pragmatic competence, register, symbolic language, style, task-based teaching

Introduction

About the department and the target group

The Department of Language Education at the Faculty of Mathematics and Physics of Charles University in Prague provides education in a number of European languages, namely English, French, German, Spanish and Russian. Its main task is to prepare their students for future international cooperation, further educational prospects and scientific activities. The faculty offers education in three study programmes, which are physics, mathematics and informatics. The Faculty, being a part of Charles University, is a research institute. Its students are therefore expected to reach prominent positions in Czech research and educational institutions.

The Department of Language Education thus has to reflect these aspects and offer adequate language courses. Language training is realized in five semesters in bachelor study programmes and a two-semester postgraduate course. Undergraduates are bound to pass a comprehensive examination in English. After passing the exam, they can enrol on the courses of English for Specific Purposes (i.e. for mathematicians, physicists and information scientists). Combining language education with such specific specializations is not an easy task, considering the scientific profile of the studies. When designing the language curriculum, the specialists of the department had to deal with two main restraining aspects – the nature of the scientific language (discourse) and specific learning needs of students (probable future scientists).

Language of Science

The language of science is a very specific domain of stylistics. This particular character results from the function of scientific texts and the narrow community it is aimed at and used by. The main function is to formulate accurate, clear and relatively complete utterance (Čechová, 2008). The choice of information and the form are to have the reader create an unambiguous and entire image of the scientifically described reality. In addition, the recipient is expected not only to understand the main ideas but also to learn and cognitively process the content. It is mainly because the result of studying a scientific text is to further develop the newly acquired thoughts and apply them. Scientific texts are not intended to be read only but primarily to be studied. All the findings of theoretical stylistics must be taken into account when preparing the curriculum of the language course for academic and scientific purposes.

Linguistic features

Linguistic features of scientific texts appear to be rather rigid and stable as most of them have been observed since the beginnings of science and the development of its discourse. They remain the same even though the subject of science changes and extends dramatically. All the linguistic characteristics depend mostly on the essential criterion: the maximum parallelism between the language form and the meaning (Čechová, 2008). To meet this criterion, the language of science uses specific language means that cannot be found in any other texts.

- Mathematization
  Scientific language is dominated by facts and strictly logical argumentation. We can even observe that in various disciplines of science, the language incorporates some characteristics of the mathematical discourse (Čechová, 2008).
- Depersonalisation
  Personal contribution towards the scientific topic is often depersonalized (e.g. by means of plural or impersonal constructions).
- Low emotional charge
  Together with hiding the author's personality, scientific texts feature low emotional charge. The author avoids showing any personal attitude towards the topic for the sake of objectivity. As Čechová (2008) claims, purely standard code is used in scientific texts as it prevents authors from expressing ideas with emotional tone.
- Composition
  Paragraphs in scientific texts are very cohesive, dealing with a particular idea only. All sentences are logically combined (not necessarily by means of connectors) to induce or deduce new ideas.
- Syntax
  Syntactic construction of scientific texts reflects its mental complexity. Thus, sentences in linguistic texts are longer and more complex (the average number of words in paratactic and hypotactic clauses in Czech scientific texts accounts for almost 20 words, combined in 4 or 5 sentences). Hypotactic sentences prevail in written texts in which sentences are also longer (Čechová, 2008).
- Morphology
  Some forms prevail in scientific texts in comparison with other texts. It is the use of present and future verb forms. Among parts of speech, it is a repertoire of connectors, nouns and adjectives and a number of prepositional phrases (Čechová).
- Lexis
  Nouns are used technically in the form of scientific terminology (which is often based on words of Latin or Greek origin). Low proportion of synonymy among the words used in scientific texts reflects the tendency to avoid ambiguity. Consequently, the range of lexis suffers from monotonicity in scientific texts. Understanding and using terms properly is more important for the author and the recipient than the variety of the expressions used (low expressivity). Urbanová (2008) asserts that expressiveness and matter-of-factness cannot be that easily separated since they concur in fulfilling the communicative purpose of texts and utterances.
- Thematic progression
  Objective word order prevails in the sentences of scientific texts. Information is developed by means of thematization of rhemes.
- Text graphics
  Because of the complexity of scientific texts, graphical means are used to make them comprehensible (system of brackets, lettering, numbering, overuse of punctuation, symbols and other signs).
- Language economy
  Authors of scientific texts condense all information to be as precise as possible. Fewer words are used to avoid redundant information. Sentences are therefore condensed by means of non-finite verbs forms.

Results and activity as an inseparable part of scientific texts

As we have stated above, in science texts do not fulfil only the informative function. Mlíkovská (1977) claims that *"the content of language is formed under the influence of the circumstances of usage, the difference, however, being that in science this process is predominantly organized and goal-directed: it is to help to obtain, fix and deduce products of this specific human activity – products of a cognitive character."* This specific function of scientific texts must be observed when developing students' reading comprehension in English for academic and scientific purposes classes. Comprehension cannot be acquired and later tested as simple understanding general or specific information in a text. In the field of science, this concept of comprehension might lead to formalism. Especially in mathematics, understanding a text is a synonym for understanding a particular theory, which can only by demonstrated by human activity, e.g. solving scientific (mathematical) problems. Mlíkovská (1977) tries to specify the whole process of creating a text and its functioning in science distinguishing five stages:

1. The speaker or author finds and creates
2. a concrete mode of linguistic expression of

3. a set of thoughts, of information on the empirical or theoretical level, in order to
4. ensure adequate understanding of the recipient or reader
5. **the effect of understanding being shown by certain specialized activities of the recipient, the final effects of which manifest themselves not only in the mind of the recipient, but, first and foremost, outwardly.**

When designing a specialized course of English for mathematicians, one should necessarily consider the different use and purpose of the language of mathematics (not English necessarily). Let us make a short outline of the particularities that a 'mathematical English' is specific for.

Specifics of Mathematical Texts

The relation of mathematics and mathematical texts is closer than that of biology and biological texts. As Nebeský (1977) states, text in mathematics has the same functions as experiments, measurements and collecting or interpreting empirical data for other sciences. This close relation is obvious from any part of a mathematical text (numerous graphemes or the structure of the text).

Symbolic language

Texts in mathematics feature the parallel use of verbal and non-verbal expressions:

There exists a natural number n such that $3 \not| \; 2^n - 1$.

All these combinations can be expressed verbally, using a subordinate clause. Still, it is inappropriate for mathematicians to avoid using mathematical formulae in such cases. Nebeský (1977) asserts that symbolic language is used in mathematics where natural, non-symbolic language appears to be unreliable to express the mathematical idea. Symbolism is simply used not only to shorten the ideas (complex thought expressed in a minimum number of symbols) but also to point out the structure of the idea. This can easily by seen in such structures that are not linear but two-dimensional (e.g. symbolic expression of matrices).

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 4 & 5 \end{pmatrix}$$

The matrix can be interpreted as a three-membered sequence of the three-dimensional vectors (1; 2; 3), (2;, 5; 6), (3; 4; 5) or a three-membered sequence of the three-dimensional vectors (1; 2; 3), (2;, 5; 4), (3; 6; 5). The symbol *det* turns this geometrical interpretation to a numerical one. The whole expression can therefore be calculated resulting in a numeric sum, which is equal to -4. Its *absolute value* then represents the *volume* of the *tetrahedron* determined by either of the two sequences of the three vectors. The symbolic representation therefore structures the idea, gives it a spatial model, which can be interpreted numerically again. This comprehension is only possible because of the (cognitive) activity of the reader. This activity is not necessarily mental but also physical as the reader must use a pen and a paper to process the mathematical language into its comprehensive interpretation.

Moreover, symbolic (non-verbal) and verbal expressions are used in an extraordinary unity. Nebeský (1977) gives an example similar to the following one:

389 + lg abc

and claims: *"The dividing-line between the symbolic and the verbal would not appear to be the main dividing-line in mathematical language. This is not merely because part of the vocabulary employed in a given mathematical text is technicalized, but also because the symbolic material of a mathematical text is often not homogenous."* In the above-mentioned expression, there are three components: *sin, abc* and *274.* Each of these three-signed symbols has a different function. *389* is a decimal representation of a number, *abc* represents two algebraic multiplicative operations between three unknowns *(a, b ,c)*. The sign *lg* represents a *function*. Implicitly, the information given is much more complex. The symbol identifies not only the name of the function, but also an action that must be do ne with *abc*. In addition, the knowledgeable reader is aware of the properties of such a function (e.g. the domain, the range and the basis).

Another example in which verbal and non-verbal expression mutually coincide is the (over)use of *let*, which is usually used to “set the scene”. In other words, *let* introduces some necessary conditions to be considered in what follows:

Let $a \in R$. *Then ...*

The verbal interpretation of the mixture of verbal and symbolic expressions (*Let a be an element of the domain of real numbers.*) mutually coincides. The reader/speaker combines their content and linguistic knowledge, which is especially apparent from the use of the bare infinitive of the verb *to be*.

Lexis

Stiff and Live Expressions

Texts in mathematics are also full of fixed and live expressions (Nebeský, 1982, 1984). Fixed expressions are those whose use is properly determined, fixed or defined. Contrary to other sciences, mathematical texts are full of stiff expressions adopted from other texts (frequent intertextuality) while many other expressions become fixed only for the purpose of the text itself. The letter *n* can represent any natural number in one text, but a number of the vertices of a triangle in another. In addition, one symbol can play more functions within one text for the sake of the language economy. In such cases, expressions can be continuously redefined by the author of the text. Stiff expressions are not only symbols and longer symbolic formulations but also words and phrases. Some of them can be so widespread that they can be found in many fields of mathematics (e.g. *set, empty, or, normal*). Stiff expressions thus turned into terms. As Nebeský claims, *“It is impossible to recognize which expressions are stiff if one does not have good command of the mathematical discipline and without an overall knowledge of the text“.*

Besides stiff expressions, mathematical texts are full of live expressions. Their function is usually not clearly defined. They only help mathematicians to speak about mathematical objects (stiff expressions) or order the ideas of mathematicians properly. The use of live expressions is always highly dependent upon the stiff expressions which the text describes and the branch of mathematics. Some expressions can therefore be used as stiff in one text but as live in another. The use and interpretation of live and stiff expressions is determined by the topic, text (co-text) and convention. This peculiarity of mathematical texts is caused by the fact that mathematics deals with abstract and uniquely constructed ideas which must be expressed unambiguously by means of a natural language.

We can thus conclude that interpretation and comprehension of a mathematical text can be extremely difficult for a mathematician (without really deep knowledge of the whole text/theory) and impossible for a mathematically non-educated person. Moreover, composing a mathematical text is based on balancing the use of stiff and live expressions properly, which is a matter of real mastery. This mastery represents an indispensable competence of the author, which is unique and specific for mathematicians.

Terminology

Mathematics has a significant position among other sciences given by its strictly axiomatic structure, deductive reasoning and very precise terminology. Every term in mathematics is profoundly defined to such an extent that no ambiguities are acceptable. This tendency is only possible because mathematics anywhere in the world describes and interprets the same abstract reality, regardless of external (socially determined) influences. Surprisingly, this phenomenon does not eliminate a number of discrepancies in the mathematical terminology of two different languages. Students of mathematics are often disconcerted when they find such differences as follows:

- Non-existent lexis

The most substantial discrepancy in the system of terminology is the case of non-existent lexical unit for the same entity in either the native or the foreign language. Such terms are usually described by means of a defining term and a specifying modifier, phrase or clause, such as in the following examples:

incentre (English)- *střed kružnice vepsané (Czech) (i.e. the centre of an inscribed circle)*
circumcentre (English) - *střed kružnice opsané (Czech) (i.e. the centre of a circumscribed circle)*
or vice versa
rectangular prism (English) – kvádr (Czech)

Students of mathematics usually do not expect non-existence of equivalent terms in either of the languages.

- Non-equivalent polysemy

Some terms are polysemous in either the source or the target language, but there is no one-to-one correspondence between the range of meanings.

segment (English) – úsečka (Czech)
segment (English) – úseč (Czech)

circle (English) – kružnice (Czech)
circle (English) – kruh (Czech)

As students of mathematics expect one-to-one correspondence between the terminologies of the source and the target language, they can be disoriented and search for other forms of naming the entity.

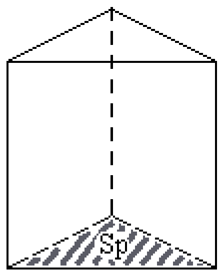
- False friends

Students of mathematics can also be confused by numerous "false friends". They usually use these expressions incorrectly in their oral or written production.

chord *(English)*	–	*těTiva (Czech)*
radical axis (English)	–	***chord****ála (Czech)*
derivation *(English)*	-	*odvození (Czech)*
differentiation (English)	-	***derivování, derivace*** *(Czech)*
derivative (English)	-	***derivace*** *(Czech)*

- Different perception of reality

Reality can be perceived differently in two different codes (Pinker, 2009). This can easily be demonstrated on the verbal expression of geometrical interpretation of reality. For instance, names of solid figures in one language are derived on the basis of different criteria:



triangular prism (English) (the bottom face is a triangle) – trojboký hranol (i.e. three-faced prism)
hexagonal prism (English) (the bottom face is a hexagon) – šestiboký hranol (i.e. six-faced prism)

All the lexical discrepancies can combine with one another as it is apparent in the following example:

circumference (English) – délka kružnice (the length of a circle) or *obvod kruhu (the perimeter of a circle)*

The word *circumference* does not have a lexical equivalent in Czech and the entity of a circle is perceived differently in Czech. The two manners of translation are therefore very difficult to explain to an English native speaker.

Grammar

Performative verbs

Mathematical texts differ from texts of other sciences in use of specific grammatical means. It is quite frequent to use the first person singular imperative forms (e.g. *Let us consider, Let us define*) or the first person plural future forms (e.g. *We will consider, We will choose*). These are usually used when indicating performance. As it has been claimed above, text is a laboratory of a mathematician and all performance must therefore be described in detail (performative hypothesis by Yule, 1996). It is a convention of mathematical text to avoid using the more traditional performative form, i.e. the first person singular or plural indicative present form.

Articles

The relation between grammatical means and the structure of mathematical texts can also be demonstrated on the use of articles. Mathematical texts are structured in such a way that one section functions as a whole (it can be a sentence). One mathematical object can therefore be named and renamed repeatedly by the same designation. This affects the use of indefinite article, which is used for each new use of the object. Such use is unnatural in fiction and texts of other sciences.

The use of articles also differs in case of objects determined by numbering and lettering. In academic grammar books, nouns combined with numbers and letters are thought to be used with no articles, such as *Room 10, Tram 210, Paragraph A.* In the mathematical (scientific) text, numbering and lettering is frequent. It is used either to structure the text (*Theorem 5: Let us...*), as a means of intratextual reference *(...as we have proved in Theorem 5.)* or to refer to objects that are being considered (*the triangle ABC*). The last example makes difficulties to students. *ABC* refers to the whole class of objects (it is not any particular triangle, there is no singular reference) but it is determined by lettering. The mathematical generality seemingly clashes with the grammatical definiteness (expressed by lettering).

Conclusion: Artificiality and Naturalness

We can thus conclude that in comparison with other specialized texts, mathematical texts can simply be labelled as artificial. As we have demonstrated, this artificial nature affects the repertoire of lexical and grammatical means used by mathematicians. It can also be demonstrated on the discourse structure of a mathematical text, which is characteristic for the sequences of definitions, theorems and proofs. Still, texts in mathematics also depend on the expressive power of natural (non-technicalized) language devices. The border between natural and artificial is not clear as natural expressions are technicalized in mathematical texts and, vice versa, artificial (technical) expressions are used naturally by mathematicians. As Nebeský (1977) states: *"The very exactness of a mathematical idea admits the undisturbed transfer of the content of the idea, even given a certain natural looseness of the means of expression employed. The exact content of an idea can be extracted and understood from a mode of expression that may well be considered somewhat inexact, unclear or incomplete."* Teaching students of mathematics to express mathematical ideas in a foreign language must take the dichotomy of artificial and natural into great consideration.

Students of mathematics as a target group

The other aspect that had to be taken into consideration when designing the curriculum of the course English for Mathematicians was the nature of students of mathematics.

First, it is their language needs that arise from the profile of the study programme they enrolled on. As we have said, students of the Faculty of Mathematics and Physics of Charles University in Prague are supposed to by theoretical physicists, mathematicians and informatics specialists. They are expected to continue in the postgraduate studies and most probably they will become scientists and researchers in these disciplines.

Second, it is necessary to consider the specific aptitudes of our students. It is highly presumable that it is mathematical and logic intelligence that dominates their intelligence distribution pattern (Gardner, 1999). The essential function of this intelligence is confrontation of a human being with the world of entities, their arrangement and organisation (Gardner, 1999). People dominated by mathematical and logic intelligence are able to estimate quantity, easily understand symbols and symbolic language, handle abstract operations. They are keen on computing and solving problems. They do experiments and are eager to deal with puzzles that they do not understand. They want to do things themselves. Folprechtová (2006) claims: *"In any school subject all types of intelligence can be developed. ... Teaching and learning a foreign language is such a complex activity that it calls for this combination of approaches and, what is more, it also offers opportunities for them from its essence."*

Besides Gardner's theory of multiple intelligence, the theory of learning styles might help us reveal specific learning needs of students of mathematics. We have therefore done research into our students learning styles, using Kolb's research method and classification. This year, we have asked a number of our students to voluntarily and anonymously fill out a questionnaire. It was Kolb's LSI II A questionnaire published at Auburn University of Montgomery in 1986 (in translation by professor Mareš, the Faculty of Medicine, Charles University in Prague, published in Turek, 2001). We received 47 properly completed questionnaires.

Kolb distinguishes four learning styles on the basis of two criteria:

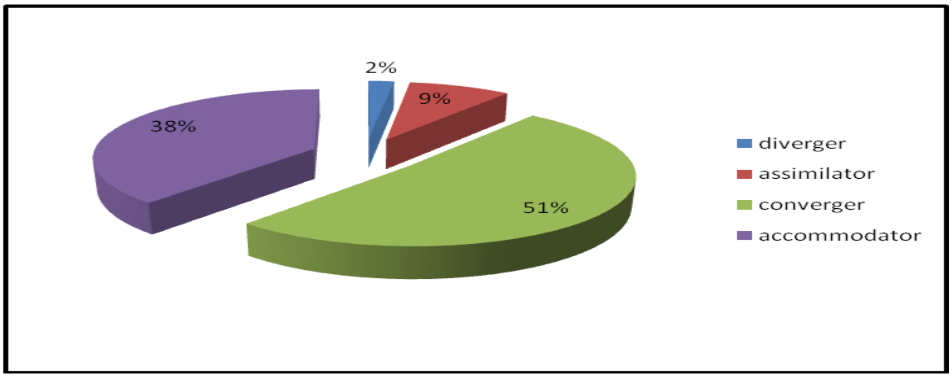
- Perception of information: in either concrete or abstract form

- Processing of information: by means of reflective observation or active manipulation

These four poles divide learning styles into four quadrants:

		concrete experience	
	dynamic (accommodator)	*innovator (diverger)*	
active experimentation			**reflective observation**
	practitioner (converger)	*analyst (assimilator)*	
		abstract conceptualisation	

Our research revealed the following distribution of our students in the four types:



It is apparent that two learning styles predominate (converger and accommodator) while the other two are rare (diverger and assimilator). The vast majority (89%) of the students who participated in the inquiry have the same approach towards learning in terms of processing new information. Since they are either accommodators or convergers, they prefer learning by active experimentation. 60% of them prefer abstract conceptualization (convergers and assimilators) and 40% privileges concrete experience (accommodators and divergers). Let us now consider the two dominant learning styles more profoundly.

Convergers (called practitioners by Kolb) perceive information in an abstract form but process them actively. They want to know how abstract concepts work in real situations. They like solving problems and apply ideas. They consider teachers as trainers who organize and manage the teaching and learning process. They are said to prefer studying applied sciences. They are introverts who are best motivated by problems as they want to know how abstract ideas function. They learn faster if they can be active, do things “with hands“ (have preference of kinaesthetic activities). The heuristic method is an effective way of teaching which they appreciate most.

Accommodators prefer gaining information in a concrete form which they process actively. They often learn new things by trial-and-error method. They are impulsive and impatient. They need to deal with problems practically since they want to discover things and ideas and to apply what they have learnt. They are best motivated if they can see the result of their hard work. They prefer cooperative and project teaching methods.

The micro-research did not prove the hypothesis entirely that the type of assimilators and convergers would prevail. As we expected it is abstract conceptualisation that is a dominant means of perceiving information. Surprisingly, 89% learn by active experimentation rather than reflective observation.

Curriculum of the course *English for Mathematicians*

When designing the curriculum of the course English for Mathematicians, we had to consider all the specifics of the learning needs of mathematicians and the peculiarities of mathematical texts and language briefly described above. We can thus summarize some prerequisites:

- The content and methods involved in the course must reflect the needs and learning style of students of mathematics. Teachers should always be aware of the particularities, focusing on developing analytical skills of the students of mathematics. Students of mathematics are capable of processing abstract ideas (content) but they appear to prefer learning by doing and applying their knowledge in experiments and problem solving tasks (methods).
- Special attention must be paid to the stylistic features of mathematical texts. Students cannot simply start developing their receptive and productive skills without explicitly dealing with the language and style of mathematical/scientific language.
- Since mathematical ideas must always be expressed precisely and unambiguously, the course must primarily focus on the development of linguistic and pragmatic competence (Hedge) to result in accuracy.
- As we have shown, in comparison with other scientific texts even the style and structure of mathematical texts are fixed and rigorous. Thus, it is also necessary to focus on developing discourse competence (Hedge). In addition, students have almost no experience with the concepts of the particular genres of scientific (mathematical) texts in English.
- Since the lexical level of mathematical texts plays a substantial part in the language of mathematics, the course should explicitly deal with mathematical terminology. Particular attention must be paid to discrepancies in the terminological systems of the national and foreign languages. Misuse of terminology in mathematics is totally unacceptable.
- Comprehension of a mathematical text in a foreign language in its complexity is to result in activity based on solving problems, explaining and deducing theories. The course of English for mathematicians should therefore be task-based oriented.

All these prerequisites are essential to premeditate as the attenders of the course English for Mathematicians usually have some experience with studying foreign languages. At primary and secondary schools in the Czech Republic, the paradigm in the theory of teaching foreign languages follows the principles of communicative approach (Larsen-Freeman, 2000). Undergraduates can therefore deal with a number of situations from their everyday life in a foreign language because their strategic competence (Hedge, 2000) has been fairly highly developed. But their language command sometimes lacks in accuracy.

Curriculum

The syllabus of the course of *English for Mathematicians* was divided into three stages, observing the above-stated prerequisites. It can be described briefly by the following table:

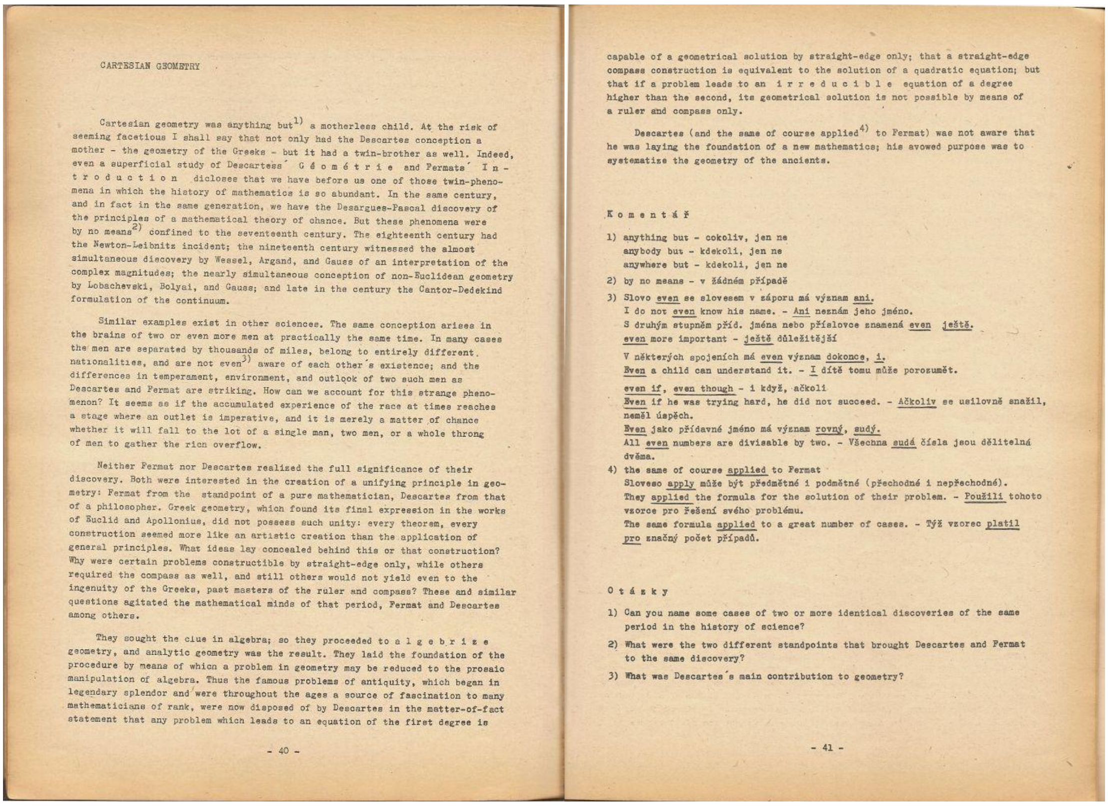
AREA	**LANGUAGE FOCUS**	**TASKS**
STAGE ONE **-** **STYLE AND STRUCTURE**	a) The language of science – discourse/text analysis, orality vs. literacy, the level of formality, formal and informal academic words and expressions b) Research articles and other research genres (abstracts, research presentations, theses and dissertations), organizing academic writing c) Presentations – language, structure and analysis, making presentations d) Symbolic language and mathematical notation, punctuation	Simple arithmetic and algebraic tasks in order to practise reading symbolic language. Each student is to prepare and present a mathematical topic. Complete analysis of a scientific text.
STAGE TWO **-**	a) Quantifying expressions b) Talking about/describing facts, evidence and data, numbers, statistics, graphs and diagrams, cause and	Simple arithmetic and algebraic tasks in order to practise reading symbolic language.

FUNCTIONS AND NOTIONS	effect c) Analysing results d) Presenting an argument e) Describing research methods f) Classifying g) Comparing and contrasting h) Defining i) Stating theorems j) Proving theorems	Formulating definitions, theorems and their proofs.
STAGE THREE - **GRAMMAR AND TERMINOLOGY**	a) Non-finite verb forms b) Passive voice c) Inversion d) Articles e) The language of geometry f) The language of algebra g) The language of analysis	Complex mathematical problems and sequences of problems from the basic disciplines of mathematics.

Stage one

In the introductory part of the course of English for Mathematicians, students deal with the stylistic properties of scientific and mainly mathematical texts (grammatical, lexical, structural and other features). Students analyse a number of texts pointing out the differences between technical and literary/popular texts. Particular attention is paid to genres that students are supposed to deal with in their further practice, namely abstracts, research papers, articles and presentations. The analysis is based on noticing and subsequent interpretation mainly.

It is important to work with texts that are natural for mathematicians. Authenticity is not the only criterion because there are authentic texts about mathematics that do not observe the naturalness of the language of mathematics. They are either written by non-mathematicians for other non-mathematicians or by mathematicians for non-mathematicians (popular texts, journalistic texts). Still, such texts lack in the purpose of goal-directed scientific activity (Mlíkovská, 1977). These texts were and sometimes are used in textbooks of English for mathematicians as we can see in the following examples:



CARTESIAN GEOMETRY

Cartesian geometry was anything but[1] a motherless child. At the risk of seeming facetious I shall say that not only had the Descartes conception a mother - the geometry of the Greeks - but it had a twin-brother as well. Indeed, even a superficial study of Descartess´ G é o m é t r i e and Fermats´ I n t r o d u c t i o n dicloses that we have before us one of those twin-phenomena in which the history of mathematics is so abundant. In the same century, and in fact in the same generation, we have the Desargues-Pascal discovery of the principles of a mathematical theory of chance. But these phenomena were by no means[2] confined to the seventeenth century. The eighteenth century had the Newton-Leibnitz incident; the nineteenth century witnessed the almost simultaneous discovery by Wessel, Argand, and Gauss of an interpretation of the complex magnitudes; the nearly simultaneous conception of non-Euclidean geometry by Lobachevski, Bolyai, and Gauss; and late in the century the Cantor-Dedekind formulation of the continuum.

Similar examples exist in other sciences. The same conception arises in the brains of two or even more men at practically the same time. In many cases the men are separated by thousands of miles, belong to entirely different nationalities, and are not even[3] aware of each other´s existence; and the differences in temperament, environment, and outlook of two such men as Descartes and Fermat are striking. How can we account for this strange phenomenon? It seems as if the accumulated experience of the race at times reaches a stage where an outlet is imperative, and it is merely a matter of chance whether it will fall to the lot of a single man, two men, or a whole throng of men to gather the rich overflow.

Neither Fermat nor Descartes realized the full significance of their discovery. Both were interested in the creation of a unifying principle in geometry: Fermat from the standpoint of a pure mathematician, Descartes from that of a philosopher. Greek geometry, which found its final expression in the works of Euclid and Apollonius, did not possess such unity: every theorem, every construction seemed more like an artistic creation than the application of general principles. What ideas lay concealed behind this or that construction? Why were certain problems constructible by straight-edge only, while others required the compass as well, and still others would not yield even to the ingenuity of the Greeks, past masters of the ruler and compass? These and similar questions agitated the mathematical minds of that period, Fermat and Descartes among others.

They sought the clue in algebra; so they proceeded to a l g e b r i z e geometry, and analytic geometry was the result. They laid the foundation of the procedure by means of which a problem in geometry may be reduced to the prosaic manipulation of algebra. Thus the famous problems of antiquity, which began in legendary splendor and were throughout the ages a source of fascination to many mathematicians of rank, were now disposed of by Descartes in the matter-of-fact statement that any problem which leads to an equation of the first degree is

- 40 -

capable of a geometrical solution by straight-edge only; that a straight-edge compass construction is equivalent to the solution of a quadratic equation; but that if a problem leads to an i r r e d u c i b l e equation of a degree higher than the second, its geometrical solution is not possible by means of a ruler and compass only.

Descartes (and the same of course applied[4] to Fermat) was not aware that he was laying the foundation of a new mathematics; his avowed purpose was to systematize the geometry of the ancients.

K o m e n t á ř

1) anything but - cokoliv, jen ne
anybody but - kdokoli, jen ne
anywhere but - kdekoli, jen ne
2) by no means - v žádném případě
3) Slovo even se slovesem v záporu má význam ani.
I do not even know his name. - Ani neznám jeho jméno.
S druhým stupněm příd. jména nebo příslovce znamená even ještě.
even more important - ještě důležitější
V některých spojeních má even význam dokonce, i.
Even a child can understand it. - I dítě tomu může porozumět.
even if, even though - i když, ačkoli
Even if he was trying hard, he did not succeed. - Ačkoliv se usilovně snažil, neměl úspěch.
Even jako přídavné jméno má význam rovný, sudý.
All even numbers are divisable by two. - Všechna sudá čísla jsou dělitelná dvěma.
4) the same of course applied to Fermat
Sloveso apply může být předmětné i podmětné (přechodné i nepřechodné).
They applied the formula for the solution of their problem. - Použili tohoto vzorce pro řešení svého problému.
The same formula applied to a great number of cases. - Týž vzorec platil pro značný počet případů.

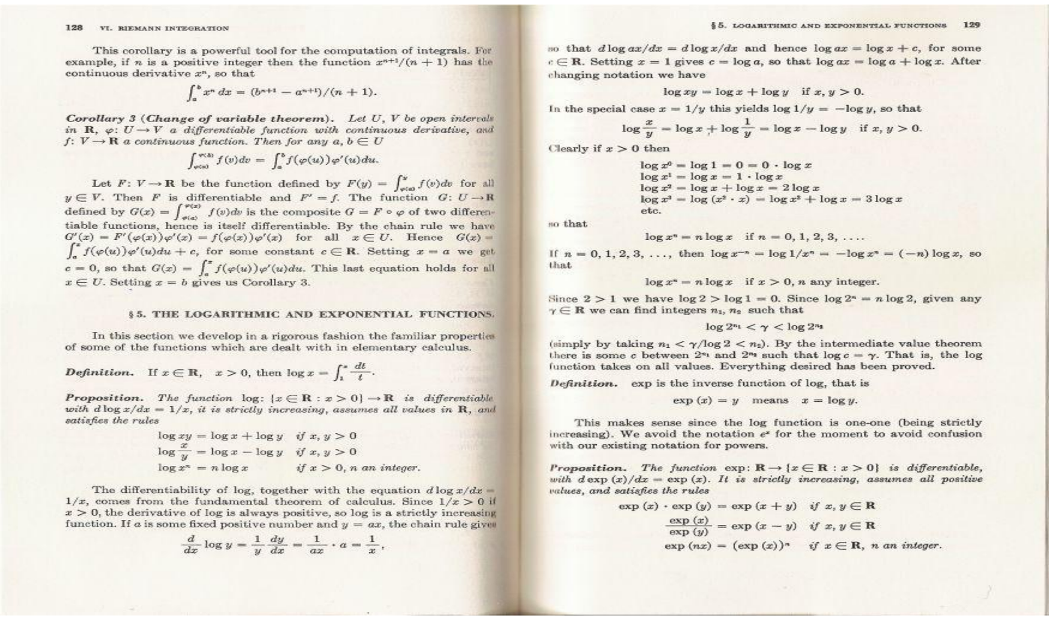
O t á z k y

1) Can you name some cases of two or more identical discoveries of the same period in the history of science?
2) What were the two different standpoints that brought Descartes and Fermat to the same discovery?
3) What was Descartes´s main contribution to geometry?

- 41 -

A sample text on Cartesian Geometry used in the textbook Úvod do četby anglických odborných textů pro posluchače MFF UK by Vladimír Mach.

Adequate texts used in the classes of English for mathematicians should not only be authentic but mainly natural. They must be written by mathematicians for mathematicians. Such texts can only fulfil the criterion of cognitive scientific activity as the result of comprehension. Comparison of the two kinds of natural texts reveals obvious and fundamental differences:



128 VI. RIEMANN INTEGRATION

This corollary is a powerful tool for the computation of integrals. For example, if n is a positive integer then the function $x^{n+1}/(n+1)$ has the continuous derivative x^n, so that

$$\int_a^b x^n\,dx = (b^{n+1} - a^{n+1})/(n+1).$$

Corollary 3 (Change of variable theorem). *Let U, V be open intervals in $\mathbf{R}$, $\varphi: U \to V$ a differentiable function with continuous derivative, and $f: V \to \mathbf{R}$ a continuous function. Then for any $a, b \in U$*

$$\int_{\varphi(a)}^{\varphi(b)} f(v)dv = \int_a^b f(\varphi(u))\varphi'(u)du.$$

Let $F: V \to \mathbf{R}$ be the function defined by $F(y) = \int_{\varphi(a)}^{y} f(v)dv$ for all $y \in V$. Then F is differentiable and $F' = f$. The function $G: U \to \mathbf{R}$ defined by $G(x) = \int_{\varphi(a)}^{\varphi(x)} f(v)dv$ is the composite $G = F \circ \varphi$ of two differentiable functions, hence is itself differentiable. By the chain rule we have $G'(x) = F'(\varphi(x))\varphi'(x) = f(\varphi(x))\varphi'(x)$ for all $x \in U$. Hence $G(x) = \int_a^x f(\varphi(u))\varphi'(u)du + c$, for some constant $c \in \mathbf{R}$. Setting $x = a$ we get $c = 0$, so that $G(x) = \int_a^x f(\varphi(u))\varphi'(u)du$. This last equation holds for all $x \in U$. Setting $x = b$ gives us Corollary 3.

§ 5. THE LOGARITHMIC AND EXPONENTIAL FUNCTIONS.

In this section we develop in a rigorous fashion the familiar properties of some of the functions which are dealt with in elementary calculus.

Definition. If $x \in \mathbf{R}$, $x > 0$, then $\log x = \int_1^x \frac{dt}{t}$.

Proposition. *The function* $\log: \{x \in \mathbf{R} : x > 0\} \to \mathbf{R}$ *is differentiable with $d \log x/dx = 1/x$, it is strictly increasing, assumes all values in $\mathbf{R}$, and satisfies the rules*

$$\log xy = \log x + \log y \quad \text{if } x, y > 0$$
$$\log \frac{x}{y} = \log x - \log y \quad \text{if } x, y > 0$$
$$\log x^n = n \log x \quad \text{if } x > 0, n \text{ an integer.}$$

The differentiability of log, together with the equation $d \log x/dx = 1/x$, comes from the fundamental theorem of calculus. Since $1/x > 0$ if $x > 0$, the derivative of log is always positive, so log is a strictly increasing function. If a is some fixed positive number and $y = ax$, the chain rule gives

$$\frac{d}{dx}\log y = \frac{1}{y}\frac{dy}{dx} = \frac{1}{ax}\cdot a = \frac{1}{x},$$

§ 5. LOGARITHMIC AND EXPONENTIAL FUNCTIONS 129

so that $d \log ax/dx = d \log x/dx$ and hence $\log ax = \log x + c$, for some $c \in \mathbf{R}$. Setting $x = 1$ gives $c = \log a$, so that $\log ax = \log a + \log x$. After changing notation we have

$$\log xy = \log x + \log y \quad \text{if } x, y > 0.$$

In the special case $x = 1/y$ this yields $\log 1/y = -\log y$, so that

$$\log \frac{x}{y} = \log x + \log \frac{1}{y} = \log x - \log y \quad \text{if } x, y > 0.$$

Clearly if $x > 0$ then

$$\log x^0 = \log 1 = 0 = 0 \cdot \log x$$
$$\log x^1 = \log x = 1 \cdot \log x$$
$$\log x^2 = \log x + \log x = 2 \log x$$
$$\log x^3 = \log (x^2 \cdot x) = \log x^2 + \log x = 3 \log x$$
etc.

so that

$$\log x^n = n \log x \quad \text{if } n = 0, 1, 2, 3, \ldots.$$

If $n = 0, 1, 2, 3, \ldots$, then $\log x^{-n} = \log 1/x^n = -\log x^n = (-n) \log x$, so that

$$\log x^n = n \log x \quad \text{if } x > 0, n \text{ any integer.}$$

Since $2 > 1$ we have $\log 2 > \log 1 = 0$. Since $\log 2^n = n \log 2$, given any $\gamma \in \mathbf{R}$ we can find integers n_1, n_2 such that

$$\log 2^{n_1} < \gamma < \log 2^{n_2}$$

(simply by taking $n_1 < \gamma/\log 2 < n_2$). By the intermediate value theorem there is some c between 2^{n_1} and 2^{n_2} such that $\log c = \gamma$. That is, the log function takes on all values. Everything desired has been proved.

Definition. exp is the inverse function of log, that is

$$\exp (x) = y \quad \text{means} \quad x = \log y.$$

This makes sense since the log function is one-one (being strictly increasing). We avoid the notation e^x for the moment to avoid confusion with our existing notation for powers.

Proposition. *The function* $\exp: \mathbf{R} \to \{x \in \mathbf{R} : x > 0\}$ *is differentiable, with $d \exp (x)/dx = \exp (x)$. It is strictly increasing, assumes all positive values, and satisfies the rules*

$$\exp (x) \cdot \exp (y) = \exp (x + y) \quad \text{if } x, y \in \mathbf{R}$$
$$\frac{\exp (x)}{\exp (y)} = \exp (x - y) \quad \text{if } x, y \in \mathbf{R}$$
$$\exp (nx) = (\exp (x))^n \quad \text{if } x \in \mathbf{R}, n \text{ an integer.}$$

A mathematical text on Logarithmic and Exponential Function from the book Introduction to Analysis by Maxwell Rosenlicht

Besides studying the specifics of the style and structure of natural texts, students must learn the verbal interpretation of the symbolic mathematical language. Although such knowledge is not essential for comprehension, it is simply necessary for reading the texts. The teacher should therefore assign tasks like these:

- Verbal interpretations and reinterpretations

  Read the following expression.

  $\{x \in X : P(x)\}$

- Calculations

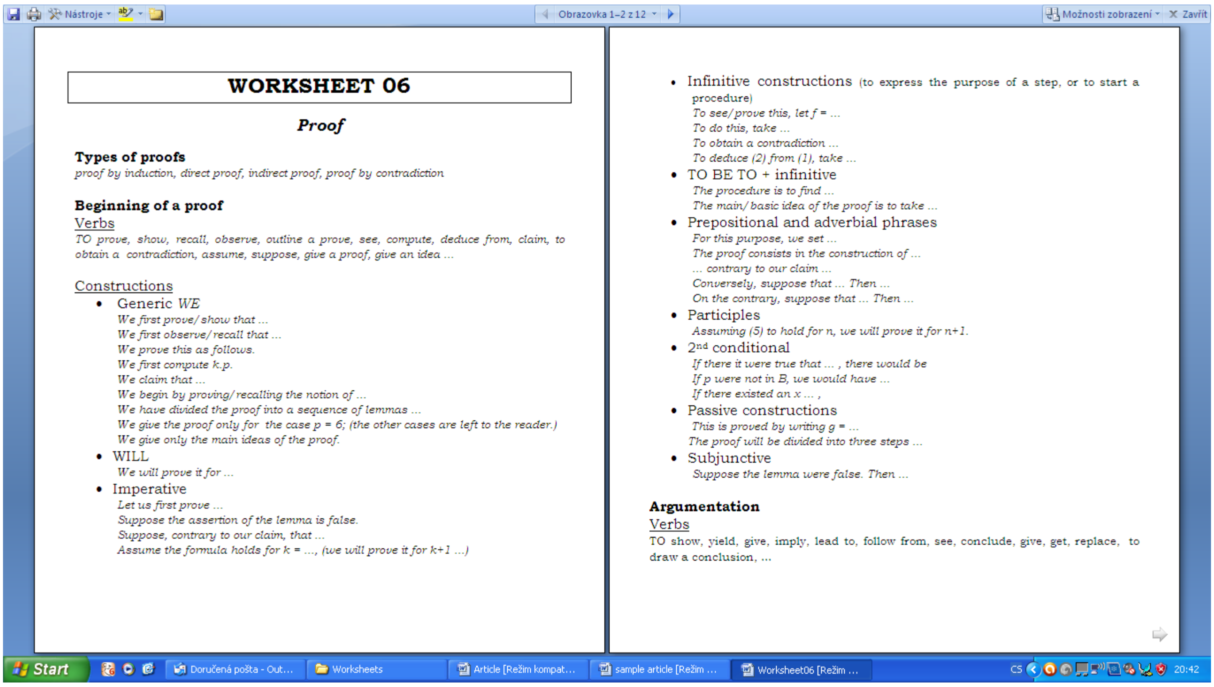
  Find the result.

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Students can even assign such tasks to each other, dictating verbal expressions for their partners to write them down symbolically. Only then can they appreciate indispensable command of the symbolic language and its verbal expression.

Stage two

In the second stage, students learn to use grammatical and lexical means in order to express basic functions and notions. It is crucial for mathematicians to correctly formulate mainly definitions, theorems and proofs.



WORKSHEET 06

Proof

Types of proofs
proof by induction, direct proof, indirect proof, proof by contradiction

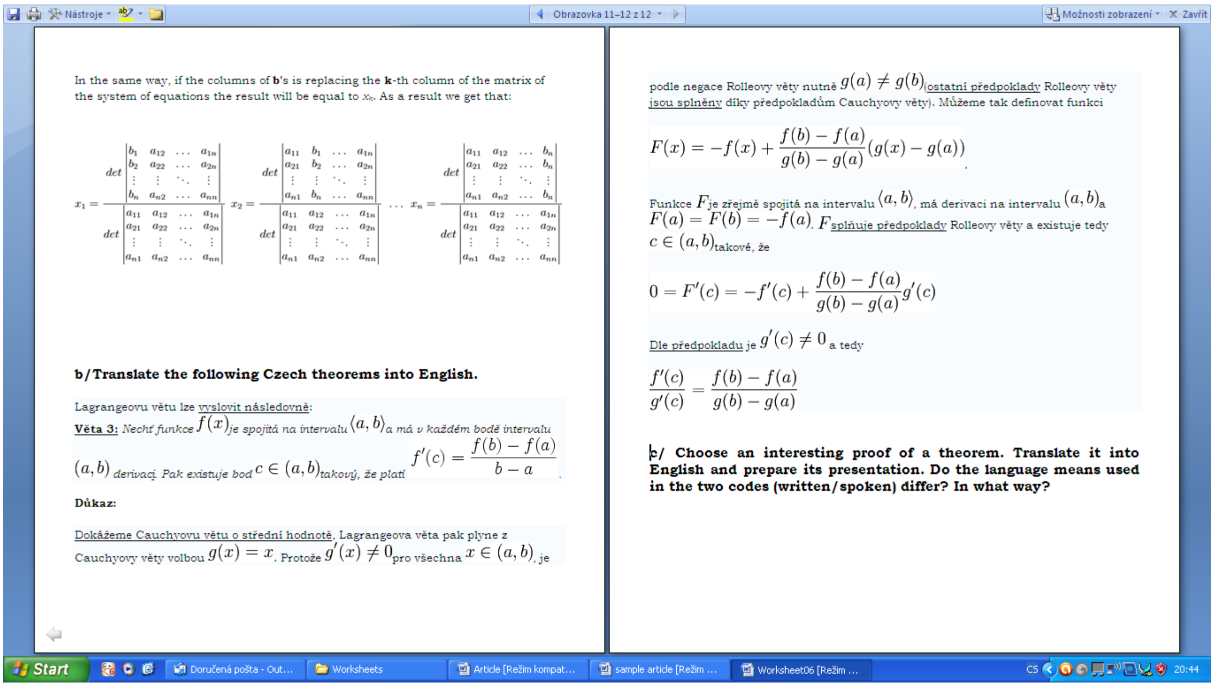
Beginning of a proof
Verbs
TO prove, show, recall, observe, outline a prove, see, compute, deduce from, claim, to obtain a contradiction, assume, suppose, give a proof, give an idea ...

Constructions
- Generic *WE*
  We first prove/show that ...
  We first observe/recall that ...
  We prove this as follows.
  We first compute k.p.
  We claim that ...
  We begin by proving/recalling the notion of ...
  We have divided the proof into a sequence of lemmas ...
  We give the proof only for the case p = 6; (the other cases are left to the reader.)
  We give only the main ideas of the proof.
- WILL
  We will prove it for ...
- Imperative
  Let us first prove ...
  Suppose the assertion of the lemma is false.
  Suppose, contrary to our claim, that ...
  Assume the formula holds for k = ..., (we will prove it for k+1 ...)
- Infinitive constructions (to express the purpose of a step, or to start a procedure)
  To see/prove this, let f = ...
  To do this, take ...
  To obtain a contradiction ...
  To deduce (2) from (1), take ...
- TO BE TO + infinitive
  The procedure is to find ...
  The main/basic idea of the proof is to take ...
- Prepositional and adverbial phrases
  For this purpose, we set ...
  The proof consists in the construction of ...
  ... contrary to our claim ...
  Conversely, suppose that ... Then ...
  On the contrary, suppose that ... Then ...
- Participles
  Assuming (5) to hold for n, we will prove it for n+1.
- 2nd conditional
  If there it were true that ... , there would be
  If p were not in B, we would have ...
  If there existed an x ... ,
- Passive constructions
  This is proved by writing g = ...
  The proof will be divided into three steps ...
- Subjunctive
  Suppose the lemma were false. Then ...

Argumentation
Verbs
TO show, yield, give, imply, lead to, follow from, see, conclude, give, get, replace, to draw a conclusion, ...

First of all, students are exposed to language forms which are sorted according to the criteria that were formulated in Stage One (e.g. first person plural, *Let*, overuse of the passive). Students study such language forms to be able to formulate definitions, theorems and proofs on their own. All the examples were excerpted from original (authentic and natural) English mathematical texts and are distributed in the form of worksheets weekly.

Finally, the stage of language practice comes, in which purely linguistic assignments and mathematical problems are combined. When selecting the methods of practice, we took into consideration the specifics of our students' learning style and the characteristic combination of stiff and live expressions in mathematical texts.



In the same way, if the columns of **b**'s is replacing the **k**-th column of the matrix of the system of equations the result will be equal to x_k. As a result we get that:

$$x_1 = \frac{det\begin{vmatrix} b_1 & a_{12} & \dots & a_{1n} \\ b_2 & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \dots & a_{nn}\end{vmatrix}}{det\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn}\end{vmatrix}} \quad x_2 = \frac{det\begin{vmatrix} a_{11} & b_1 & \dots & a_{1n} \\ a_{21} & b_2 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & b_n & \dots & a_{nn}\end{vmatrix}}{det\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn}\end{vmatrix}} \quad \dots \quad x_n = \frac{det\begin{vmatrix} a_{11} & a_{12} & \dots & b_n \\ a_{21} & a_{22} & \dots & b_n \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & b_n\end{vmatrix}}{det\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn}\end{vmatrix}}$$

b/Translate the following Czech theorems into English.

Lagrangeovu větu lze vyslovit následovně:
Věta 3: *Nechť funkce* $f(x)$ *je spojitá na intervalu* $\langle a, b\rangle$ *a má v každém bodě intervalu* (a, b) *derivaci. Pak existuje bod* $c \in (a, b)$ *takový, že platí* $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Důkaz:

Dokážeme Cauchyovu větu o střední hodnotě, Lagrangeova věta pak plyne z Cauchyovy věty volbou $g(x) = x$. Protože $g'(x) \neq 0$ pro všechna $x \in (a, b)$, je podle negace Rolleovy věty nutně $g(a) \neq g(b)$ (ostatní předpoklady Rolleovy věty jsou splněny díky předpokladům Cauchyovy věty). Můžeme tak definovat funkci

$$F(x) = -f(x) + \frac{f(b) - f(a)}{g(b) - g(a)}(g(x) - g(a)).$$

Funkce F je zřejmě spojitá na intervalu $\langle a, b\rangle$, má derivaci na intervalu (a, b) a $F(a) = F(b) = -f(a)$. F splňuje předpoklady Rolleovy věty a existuje tedy $c \in (a, b)$ takové, že

$$0 = F'(c) = -f'(c) + \frac{f(b) - f(a)}{g(b) - g(a)}g'(c)$$

Dle předpokladu je $g'(c) \neq 0$ a tedy

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

c/ Choose an interesting proof of a theorem. Translate it into English and prepare its presentation. Do the language means used in the two codes (written/spoken) differ? In what way?

Therefore, the assignment types include:
- contrastive analysis
  Students are to compare forms used in Czech and English, focusing on equivalent counterparts of Czech and English sentences.

Věta 6.3 je <u>kritériem pro rozhodování</u> o konvergenci posloupnosti.
Theorem 6.3 <u>provides criterion for</u> deciding on convergence of the sequence.

- translation into Czech
  Students are asked to translate some English sentences into Czech, paying special attention to the forms underlined.

*<u>First we prove the theorem</u> when n = 1, in which case the ordering on **R** can be put to good use. Indeed <u>we have the following result</u>.*

- gap-filling
  Students are required to fill in gaps extracted from authentic sentences.

___________ the rank of the matrix A is less ___________ the number of columns in A (r<k), then the columns of A are linearly dependent. ___________ the rank of A equals the number of columns in A (r=k), ___________ the columns of A are ___________.

- translation into English
  Students are ready to translate selected Czech mathematical sentences (definitions, theorems, proofs) into English. They are asked to pay attention to the phrases underlined.

Lagrangeovu větu lze <u>vyslovit následovně</u>:
<u>Věta 3:</u> *Nechť funkce* $f(x)$ *je spojitá na intervalu* $\langle a, b\rangle$ *a má v každém bodě intervalu* (a, b) *derivaci.*
Pak existuje bod $c \in (a, b)$ *takový, že platí* $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

- production
  Finally, students have to produce their own mathematical texts (definitions, theorems, proofs).

All tasks are designed to emphasize structures that are natural (stiff and live according to Nebeský, 1982, 1984) in English mathematical texts. Students' written and oral production is therefore stimulated to be not only acceptable and grammatical but also natural.

Some activities can be organized as games. Students define mathematical terms while the others guess. If the teacher wants to direct students' attention to some terms only, they can put the desirable terms on the board or distribute them on a piece of paper. In case of proofs, students are activated by the task to present their proofs on the board. Geometrical proofs of algebraic theorems are especially appreciated.

Stage three

In the final stage, all the work in the language classroom is framed by particular mathematical disciplines. In a semester course, we usually manage to deal with three essential fields, i.e. algebra, geometry and mathematical analysis. Students are assigned to focus on terms whose meaning can easily be transferred from Czech. Students use the language means acquired in the second stage of the course (defining).

Define the following terms:
isomorphism, algebraically closed field

Particular attention if subsequently paid to terms which students are expected not to be able to define. These include lexical discrepancies described above.

Finally, students deal with various mathematical problems from the selected mathematical disciplines. In this phase, students have to use all the language from the previous stages. They have to prove some theorems, do calculations and solve problems.

Prove the following theorems:

Let k be a circle and AB a chord of the circle k. Then the central angle over AB with respect to k equals two angles over AB at the circumference of k.

Solve the following problems:
Find the radical axis of two arbitrary intersecting/nonintersecting circles. Describe the construction.

Students are also ready to present their results to the others and compare other students' approaches towards these tasks. At the same time, they are expected to sound natural in terms of the language they use.

Conclusion: Task-based approach

As we have shown, the whole curriculum is task-based oriented. Since the very beginning, students are exposed to language by means of authentic and natural mathematical texts. Their learning always leads to activity typical of workers (researchers) in mathematics.

The whole course proceeds from the focus on language (style, structure, functions, grammar and lexis) towards problem-solving in some mathematical disciplines. What is more, this task-based orientation is two-dimensional. Each lesson in the course of English for Mathematicians starts with language focus (e.g. notation in Stage One, language used to give definitions in Stage Two, lexis in Stage Three) and moves gradually to activity in the form of mathematical tasks. At the same time, the complexity of language as well as tasks is successively upgraded.

The use of symbolic language as well as terminology and functional language is mutually combined and included in tasks. In the following example, the symbolic definition (STAGE ONE) of the limit is to be expressed by means of the functional exponents used for defining (STAGE TWO) and finally explained within the frame of the theory of mathematical analysis (STAGE THREE):

Write, read and explain the following definition:

$$\lim_{x \to a} f(x) = A \overset{def.}{\Leftrightarrow} \forall \varepsilon > 0 \exists \delta > 0 \forall \mathrm{x}_0 \in D_f : x \in P_\delta(a) \Rightarrow f(x) \in U_\varepsilon(A)$$

Simple calculations gradually turn into more complex problems. Finally, isolated problems turn into sequences of interrelated problems (Willis, 2007) towards the end of the course. In the following examples, *c* follows form *b* which is deduced from *a*.

a) Let *k* be a circle and *AB* a chord of the circle *k*. Then the central angle over *AB* with respect to *k* equals two angles over *AB* at the circumference of *k*.

b) Let *k* be a circle and *A* a point lying out of the circle *k*. Let *p, p'* be lines passing through *A* intersecting the circle *k* at points *A, B* $\in$ *p*, A', B' $\in$ *p'*. Then the following equality is true:
$|\mathrm{AB}| = |\mathrm{A'B'}|$

c) The set of all points whose power with respect to a given circle is equal is a straight line.

The process of proceeding towards the proof of theorem *c* simulates the desired mathematical research activity.

Conclusions and Recommendations

Courses of English for specific purposes are definitely unique not only in their content but also teaching approaches, methods, materials and aids used. But courses of English for mathematicians differ in a greater extent. First, the many specifics of the language of mathematics and stylistics of mathematical texts require explicit knowledge of these features. Students have usually very poor knowledge of the peculiarities from their previous studies. As communicatively-oriented language classes place emphasis on development of discourse and strategic competence, students' communication in English is fairly fluent but sometimes lacks in precision and accuracy. The course English for Mathematicians should therefore focus on developing linguistic and pragmatic competence, paying particular attention to the style, genres, their structures, and lexical and grammatical features of this register. In addition, understanding a mathematical (scientific) text does not result only in comprehension based on reinterpreting general or specific information included in the text. It is special mathematical (scientific) activity that proves thorough understanding. This goal and activity-oriented aspect of mathematical texts correspondents with the nature and prevailing learning style of students of mathematics, physics and informatics. Although they perceive new information on the basis of abstract conceptualisation, they mostly process what they have learnt by means of active experimentations. To meet these learning needs, the course of English for mathematicians should therefore deal with applications of the theoretic language and content basis in real mathematical activity. To sum up, the course activities should always result in task-based assignments, using heuristic teaching methods such as problem-solving tasks (ideally mathematical).

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